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Monopole gravitational waves from relativistic fireballs driving gamma-ray bursts

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ABSTRACT

Einstein's General Relativity predicts that pressure, in general stresses, play a similar role to energy density, $\epsilon = \rho c^2$ (with ρ being corresponding mass density), in generating gravity. The source of gravitational field, the active gravitational mass density, sometimes referred to as Whittaker's mass density, is $\rho_{grav} = \rho + 3p/c^2$, where p is pressure in the case of an ideal fluid. Whittaker's mass is not conserved, hence its changes can propagate as monopole gravitational waves. Such waves can be generated only by astrophysical sources with varying gravitational mass. Here we show that relativistic fireballs, considered in modelling gamma-ray burst phenomena, are likely to radiate monopole gravitational waves from high-pressure plasma with varying Whittaker's mass. Also, ejection of a significant amount of initial mass-energy of the progenitor contributes to the monopole gravitational radiation. We identify monopole waves with $h^{11} + h^{22}$ waves of Eddington's classification which propagate (in the z-direction) together with the energy carried by massless fields. We show that the monopole waves satisfy Einstein's equations, with a common stress-energy tensor for massless fields. The polarization mode of monopole waves is Φ_{22}, i.e. these are perpendicular waves which induce changes of the radius of a circle of test particles only (breathing mode). The astrophysical importance of monopole gravitational waves is discussed.

Key words: gravitational waves – gamma-rays: bursts

1 INTRODUCTION

The coupling of pressure (stresses) to gravity follows from Einstein's field equation, $R_{\mu\nu} - g_{\mu\nu}R/2 = 8\pi G T_{\mu\nu}/c^4$, where the left-hand side (the Ricci tensor, $R_{\mu\nu}$, the metric tensor, $g_{\mu\nu}$, and the curvature scalar, R) describes the geometry of space-time, and the right-hand side describes the energy density and stresses of the matter distribution in terms of the energy momentum tensor. For an ideal fluid, the latter is diagonal in the rest frame of the fluid, $T_{\mu\nu} = diag(\rho c^2, p, p, p)$. Whittaker (1935) has shown that the source of gravitational field is $\rho_{grav} = \rho + \Sigma_i T^{ii}/c^2$, where the latter term is $3p/c^2$ for an ideal fluid. Under local conditions in the Solar system, mass density always dominates, $\rho >> p/c^2$, and the pressure effects are not noticed. The gravitational role of pressure is perhaps the greatest in cosmology (Peebles 1993), where the evolution of the scalefactor of the Universe, $a(t)$, is governed by Whittaker's mass density, $3\ddot{a}/a = -4\pi G(\rho + 3p/c^2)$. In cosmology, pressure and energy density are comparable, and sometimes it is pressure that dominates, as is believed to happen presently in our Universe (albeit with negative pressure)(Garnavich et al. 1998). Tolman (1934) in his classic book pointed out that the gravitational attraction of the photon gas is twice as big as the gravity of matter of the same total energy density. This conclusion follows from Whittaker's formula for the photon gas with pressure $p = \rho c^2/3$.

2 PRESSURE AS A SOURCE OF GRAVITY OF RELATIVISTIC FIREBALLS

The high-pressure regime of General Relativity has not been considered thoroughly yet (Carlip 1998). Here we study the gravitational role of pressure for astrophysical objects with weak gravitational field. In order for pressure to play any noticeable role, the system under consideration should obey a relativistic equation of state, i.e. the pressure should be comparable to the energy density, as e.g. for the photon gas. In such a case, all diagonal components of the stress-energy tensor are of the same order, $T_{00} \approx 3T_{11} \approx 3T_{22} \approx 3T_{33}$. The most promising astrophysical system in this regard seems to be a high-temperature thermal pair plasma, e^+, e^-, γ,

invoked in models of gamma-ray bursts (Paczyński 1986, Goodman 1986). However, the relativistic equation of state is only a necessary, but not sufficient, condition for pressure to contribute to Whittaker's active gravitational mass density. In massive stars, very hot pair plasma comprises the inner core but its presence is not expected to affect the gravitational mass. This is because stars are in hydrostatic equilibrium and the pressure gradient is exactly balanced by gravitational forces. The virial theorem ensures that the integral of the sum of pressure and gravitational stresses vanishes, $\int(3p+\Sigma_i T^{ii}_{grav})dV = 0$, as the gravitational stress contribution is $\int \Sigma_i T^{ii}_{grav} dV = E_{grav}$ and $3\int p dV = -E_{grav}$. As the net contribution of stresses vanishes, the active gravitational mass is the total energy divided by c^2, in spite of very high pressure in stars. As shown by Landau & Lifshitz (1975), the virial theorem is fulfilled also in a more general situation, namely for any system executing bounded motion. From conservation of the stress-energy tensor, which in the weak-field limit of General Relativity reads, $\partial_\mu T^{\mu\nu} = 0$, it follows that $\int \partial_0 T^{0i} dV = -\int \partial_k T^{ki} dV = -\oint T^{ki} d\sigma_k = 0$ as the last surface integral vanishes by virtue of the finite extent of the system. Multiplying the conservation equation by x_k and integrating over the volume we find, after time-averaging, $\int \Sigma_i (T^{ii}_s + T^{ii}_b) dV = 0$, where T^{ii}_s and T^{ii}_b are, respectively, diagonal stresses due to all particles (including radiation) in the system and due to "walls" constraining their motion.

This somewhat abstract discussion shows that gravitational effects of pressure could show up only for systems not obeying the virial theorem. Physically, one should consider non-stationary systems with unbalanced stresses, such as e.g. relativistic pair-plasma fireballs. In recent years, astrophysicists studied such fireballs intensively, such as in models of gamma-ray bursts (Kobayashi, Piran & Sari 1999). We consider here fireballs of spherical symmetry. Promising models of gamma-ray bursts assume as the initial process the quick deposition of a huge amount of electromagnetic energy in a small volume in the form of thermal pair plasma. The fireball is also loaded with some amount of baryons. High temperature plasma forms a fluid as it is opaque to photons as a result of Thomson scattering off electrons and positrons. The fireball starts to expand slowly at $t = 0$ and then accelerates, expanding at a still higher rate. Meanwhile the temperature of the plasma drops and accelerated expansion ceases when the temperature is low enough for the e^+, e^- pairs to annihilate. According to the estimate given by Goodman (1986), plasma becomes transparent to photons at $t = t_t$ when the termal energy is $k_B T/m_e c^2 \approx 0.03 - 0.05$. The pressure drops very fast and vanishes when the photons can freely escape.

For fireballs modelling gamma-ray bursts, some baryon load is required. The acceleration time, t_z, depends on the initial energy per baryon, referred to as the random Lorentz factor of baryons in the fireball, η (Kobayashi et al. 1999), $t_z = \eta R_0/c$. At time t_z the fireball enters a coasting phase. For baryon-deficient fireballs, $\eta >> 1$ and $t_z \approx t_t$. Further expansion and interactions with the interstellar medium, while crucial for producing the observed gamma rays, will not concern us here. The gravitational pressure effects occur in the early acceleration phase.

In the following we discuss for simplicity baryon-free (i.e. pure radiation) fireballs, as the presence of baryons does not affect our conclusions, and we disregard any surrouding medium, assuming fireballs to be in empty space. Assuming the initial fireball state to be a uniform plasma at rest with initial energy density $T_{00} = \rho_{in} c^2$, the initial pressure is $p_{in} = \rho_{in} c^2/3 = T_{ii}, i = 1, 2, 3$. The active gravitating mass of the fireball at $t = 0$ is thus $M_{grav}(0) = V_{in}(\rho_{in} + 3p_{in}/c^2) = M_\gamma + M_p \approx 2M_\gamma$, where $M_\gamma = V_{in}\rho_{in}$ and $M_p \approx V_{in}\rho_{in}$ are, respectively, the mass of plasma in the fireball of initial volume V_{in}, and the pressure contribution to the gravitational mass.

The gain of a significant amount of active gravitational mass during the formation period is a direct consequence of Whittaker's formula. It is the pressure-generated contribution that grows rapidly and eventually levels off. The other contribution to the gravitational mass is provided by the total energy of the fireball, which, as a conserved quantity, remains unchanged. Before the formation of the fireball this energy is included in the progenitor mass. Hence the gravitational mass of the fireball, composed equally of energy density and pressure contributions, is not a conserved quantity. This has profound consequences as it implies emission of monopole gravitational waves. We parametrize the changing pressure contribution to Whittaker's mass as $M_p = M_\gamma F_p(t)$, where the function $F_p(t) \geqslant 0$ reflects the time evolution of diagonal components of the stress-energy tensor of the fireball.

3 MONOPOLE GRAVITATIONAL WAVES RESULTING FROM AN ELECTROMAGNETIC ENERGY BURST

The effects of non-conservation of pressure-generated gravitational mass, M_p, are accompanied by gravitational effects of the mass-energy ejection of the fireball resulting from vigorous expansion. At the end of the acceleration period the fireball in the observer's frame forms a thin spherical shell of pure radiation expanding with the velocity of light c. The thickness of the shell is about the initial radius of the fireball, R_0 (Kobayashi et al. 1999). For an observer at a distance r from the centre of the fireball, the total energy inside the sphere of radius r changes when the radiation debris leaves the sphere. This change of gravitational mass propagates as a spherical wave. We denote the mass of the progenitor of the fireball by M_0. The energy of the shell is $E_{shell} = M_\gamma c^2$ and this energy flows out of the sphere of radius r in time $\Delta t = R_0/c$. We can parametrize the change of mass-energy in the sphere of radius r as a function of time in the form $\Delta M = M_\gamma F_m(t - r/c)$, since the shell is a pure electromagnetic impulse. The change of mass is $\Delta M = 0$ for $t - r/c < -\Delta t$, and $\Delta M = -M_\gamma$ for $t - r/c > \Delta t$, and thus we have $F_m(\tau) = 0$ for $\tau < -\Delta t$, and $F_m(\tau) = -1$ for $\tau > \Delta t$. Since the passage time,Δt, is of the order of 1 s, we can approximate $F_m(\tau) \approx -\Theta(\tau)$, where $\Theta(\tau) = 1$ for $\tau > 0$ and zero otherwise. Let us note that $F_m(\tau) \leqslant 0$.

The gravitational field at a distance r from the centre of the progenitor star is given by the Schwarzschild metric in the weak-field limit, $ds^2 = (-1 + 2GM_0/rc^2)dt^2 + (1 + 2GM_0/rc^2)dr^2 + r^2 d\Omega^2$. Birkhoff's theorem ensures that, until time $t = r/c - \Delta t$, the metric at distance r remains still the same Schwarzschild metric. When the expanding shell passes the observer at r, at time $t = r/c + \Delta t$, the metric

changes, as now the mass remaining in a sphere of radius r is $M_r = M_0 - M_\gamma$. We can write the metric in the form

$$ds^2 = (-1+\frac{2GM_0}{rc^2}+\gamma^{00})dt^2+(1+\frac{2GM_0}{rc^2}+\gamma^{ii})dr^2+r^2d\Omega^2, \quad (1)$$

where

$$\gamma^{00} = \gamma^{ii} = -\frac{2GM_\gamma}{c^2}\frac{1}{r} \quad (2)$$

are corrections due to the loss of mass. We are forced to conclude that around the time $t = r/c$ a perturbation of the metric has passed by the observer at r which somewhat "ironed out" the space-time ($M_r < M_0$). We assume that $2GM_0/rc^2 << 1$ is a small perturbation of the Minkowski metric in order to obtain the form of the wave analytically, as in the linear approximation gravitational fields can be added to one another. However, the above argument based on Birkhoff's theorem is valid for a general Schwarzschild metric. We can express the propagating metric perturbation in a simple form, $h_m^{\mu\nu}(r,t) = \delta^{\mu\nu}(2GM_\gamma/c^2)F_m(t-r/c)/r$. Here the diagonal components $h_m^{\mu\mu}$ are such that, for $t > r/c+\Delta t$, $h_m^{\mu\mu}(r,t) = \gamma^{\mu\mu}(r)$ from the metric (1). The perturbation $h_m^{\mu\nu}(r,t)$ is a spherical wave. Using isotropic coordinates and introducing new fields $\bar{h}_m^{\mu\nu} = h_m^{\mu\nu} - 1/2\eta^{\mu\nu}h_\alpha^\alpha$, we find the diagonal components to be

$$\bar{h}_m^{\mu\nu}(r,t) = \delta^{\mu 0}\delta^{\nu 0}\frac{4GM_\gamma}{c^2}\frac{F_m(t-r/c)}{r}. \quad (3)$$

The only non-zero diagonal field is $\bar{h}_m^{00}(r,t)$. It corresponds to a monopole wave resulting from sperically-symmetric ejection of a part of the mass-energy of a gravitating body in the form of an electromagnetic burst.

4 MONOPOLE GRAVITATIONAL WAVES FROM CHANGING WHITTAKER'S MASS

The shell of ejected mass-energy will be preceded by the monopole wave resulting from the change of Whittaker's mass in the formation phase of the fireball. The models of gamma-ray bursts do not specify the nature of the engine that energizes the fireball. It could be collapse of the core of a massive star to a Kerr black hole, with the initial fireball energy extracted from the rotational energy of the black hole by magnetic fields (Mac Fadyen & Woosley 1999). In any case, the engine is a massive object with mass, M_{bh}, of a few solar masses, which after formation of the fireball is assumed not to change. For actual estimates we use fireball parameters from Kobayashi et al. (1999). The initial size of the fireball is $R_0 = 300000$ km (about one light-second). Suppose that the total fireball energy deposited in this volume corresponds to 1/10 of a solar mass, $M_\gamma = 0.1M_\odot$. The mean mass density is $\rho = 1.8$ g/cm^3. This is a rather low mass density and certainly the gravity of the fireball is weak. The total gravitational mass of the fireball progenitor, including the mass of the engine, is thus $M_0 = M_{bh} + M_\gamma$, which is the active gravitational mass before the formation of the fireball. At $t = 0$, when the fireball is formed, the initial pressure contribution to the gravitational mass becomes comparable to the fireball mass, M_γ, for baryon-free fireballs. It is difficult to assess how closely this value is approached, as it depends on the formation process. However, if the formation process of the pair plasma is rapid enough for a sufficiently high density of e^+, e^- charges to be produced in the whole initial volume, the photons are trapped and initially the bulk of the plasma is essentially at rest, except of the surface layer of thickness of the order of the photon mean free path. The active gravitational mass of such a fireball at rest, from Whittaker's formula, is about $2M_\gamma$. Thus the gravitational mass of the host star and the fireball grows to $M_{grav}(0) = M_0 + M_\gamma = M_{bh} + 2M_\gamma$ on a formation time-scale. The pressure contribution reaches the maximum value at the formation, and remains later at the same level, even after the fireball ceases to accelerate at $t = t_z$. To see this let us remember that the pressure-generated mass is, for expanding fireball, given by an integral of the sum of diagonal stresses,

$$M_p = \frac{1}{c^2}\int \Sigma_i T^{ii}dV = \int [(\rho + \frac{p}{c^2})\gamma^2\frac{v^2}{c^2} + \frac{3p}{c^2}]dV, \quad (4)$$

where v and $\gamma = 1/\sqrt{1-v^2/c^2}$ are, respectively, the radial expansion velocity and the Lorentz factor of the expanding fluid element. At $t = 0, v = 0$ and $M_p = M_\gamma$. At the end of the acceleration phase, $t = t_z$, pressure vanishes, $p = 0$, and $M_p(t_z) = \int \rho\gamma^2 v^2/c^2 dV \approx \int \rho\gamma^2 dV = \int T^{00}dV/c^2 = M_\gamma$.

Whittaker's mass varies as $M_{grav}(t) = M_0 + F_p(t)M_\gamma$, where the function $F_p(t)$ is introduced to describe the evolution of the active gravitational mass of the system. By definition, $F_p(t) = 0$ for $t < 0$ as the gravitational mass is then M_0. Near $t = 0$ the function steeply grows to its maximum, $F_p(0) = 1$. Model fireball calculations (Kobayashi et al. 1999, Goodman 1986) set as their initial conditions a step-like behaviour of the function $F_p(t)$ at $t = 0$. In reality, there will be some formation time of the fireball, corresponding to steep but continuous growth of $F_p(t)$. This changing Whittaker's mass generates gravitational waves, which can be calculated from the field equation $\partial_\sigma\partial^\sigma\bar{h}_p^{\mu\nu} = -16\pi GT^{\mu\nu}/c^4$ (Wald 1984). For a spherical monopole wave one can easily obtain the solution. A general form of such a gravitational wave is

$$\bar{h}_p^{\mu\nu} = A^{\mu\nu}\frac{f_p(t-r/c)}{r}. \quad (5)$$

The spherical monopole wave (5) that satisfies the wave equation in an empty space, $T^{\mu\nu} = 0$, far from the source, is also a solution of the nonuniform wave equation

$$A^{\mu\nu}\partial_\sigma\partial^\sigma\frac{f_p(t-r/c)}{r} = T^{\mu\nu}(\mathbf{r},t), \quad (6)$$

with the source

$$T^{\mu\nu}(\mathbf{r},t) = B^{\mu\nu}M_\gamma c^2F_p(t)\delta^{(3)}(\mathbf{r}), \quad (7)$$

located at $r = 0$, where $B^{00} = 0$, $B^{\mu\nu} = 0$ for $\mu \neq \nu$, and $B^{ii} = 1/3$. We can identify the source function, $F_p(t)$, with the function describing the time evolution of the active gravitational mass of the fireball. Any change of the active gravitational mass propagates with the velocity of light, c, and hence the function $F_p(t)$ generates a spherical wave $f_p(t-r/c)/r$. In equation (6) the fireball is approximated by a point-like source with time-depended gravitational mass. We define it to be the mass within a few initial radii of the fireball $R \sim R_0$. The source of gravitational radiation switches off, $F_p = 0$, when the sum of diagonal stresses vanishes there, $\Sigma_i T^{ii} = 0$. This happens when the radiation debris leave the sphere of radius R. We estimate the duration

of the source activity to be $T \sim t_f + t_a + R_0/c$, where t_f is the formation time and t_a is the acceleration time for plasma to acquire relativistic velocities (we neglect any time-dilation effects due to motion of the fireball progenitor). After all debris from the fireball leaves the sphere R, the metric becomes that of the remnant of the star with gravitational mass $M_{grav} = M_{bh}$, corresponding to $F_m = -1$. The duration of the gravitational impulse is of the order of Δt, $T \sim R_0/c$. For R_0 used above, $\Delta t = 1s$.

The solution of the wave equation (6) for a point-like source is found to be $f_p(t-r/c) = F_p(t-r/c)$. Since the total energy is conserved, there is no time-dependent contribution to the $\mu = \nu = 0$ component of the perturbation, $\bar{h}_p^{00} = 0$. The space components are

$$\bar{h}_p^{ij}(r,t) = \frac{1}{3}\frac{4GM_\gamma}{c^2}\frac{\delta^{ij}}{r}F_p(t-r/c), i,j = 1,2,3, \tag{8}$$

and the trace is $\bar{h}_{p\alpha}^{\alpha} = (4GM_\gamma/c^2)F_p(t-r/c)/r$. The perturbation of the Schwarzschild metric due to changing pressure contribution to Whittaker's mass, $h_p^{\mu\nu}(r,t)$, is a spherical wave,

$$h_p^{00}(r,t) = \frac{2GM_\gamma}{c^2}\frac{F_p(t-r/c)}{r}, \tag{9}$$

$$h_p^{ii}(r,t) = -\frac{1}{3}\frac{2GM_\gamma}{c^2}\frac{F_p(t-r/c)}{r}, i = 1,2,3. \tag{10}$$

The total gravitational impulse is described by both pressure-generated and mass-loss components, and the resulting perturbation of the metric (1) is $\gamma^{\mu\nu} = h_p^{\mu\nu}(r,t) + h_m^{\mu\nu}(r,t)$. Let us remark, that the wave (3) obtained purely on physical grounds, is a solution of the wave equation (6) with the source function $T^{\mu\nu}(\mathbf{r},t) = \delta^{\mu 0}\delta^{\nu 0}M_\gamma c^2 F_m(t)\delta^{(3)}(\mathbf{r})$.

We wish to emphasize that non-conservation of Whittaker's active gravitational mass does not violate any conservation law. Energy and momentum are strictly conserved, as the divergence of the stress-energy tensor vanishes, $\partial_\mu T^{\mu\nu} = 0$, but stresses, such as kinetic energy and pressure, are not conserved separately. In contrast, the ejection of matter, which gives rise to $h_m^{\mu\nu}(r,t)$, conserves the energy: the gravitational mass within a given radius changes by the amount taken out by fireball debris moving out of the sphere that we consider. The latter example provides a physical proof that the wave is not an artefact which can be removed by changing gauge. One can imagine a gedanken experiment, which is perhaps more suggestive. Let us consider small quantity of hydrogen gas of mass M, in a container of negligibly small mass. The metric around this body is everywhere given by a weak-field Schwarzschild formula. Let half of the initial mass be replaced by the same amount of antihydrogen, initially separated from hydrogen. The gravity of antimatter is thought to be the same as that of matter and the metric still corresponds to the same mass M. Then at $t = 0$ the separating mechanism is switched off, with matter and antimatter annihilating each other. A fireball of pure radiation of energy Mc^2 is formed. Assuming spherical symmetry and efficient annihilation, the fireball becomes a thin shell of radiation, similar to that discussed in Section 3. Clearly, a monopole gravitational wave generated during the formation of the fireball travels together with the expanding electromagnetic shell that erases the Schwarzschild matric. The space-time, after passing the wave, becomes a flat one.

5 MONOPOLE WAVES PROPAGATING WITH RADIATION OF MASSLESS FIELDS: A UNIFIED APPROACH

The energy conservation condition

$$\frac{\partial}{\partial t}\int T^{00}dV = -\frac{1}{2}\int T_{\mu\nu}\frac{\partial h_p^{\mu\nu}}{\partial t}dV \tag{11}$$

shows that the fireball, during the pressure build-up, radiates energy by gravitational waves, irrespective of the geometry. This is a major difference in comparison with oscillating non-relativistic sources, where spherically-symmetric motion does not generate any gravitational radiation. However, to find the nature of emitted waves we must first consider the question of coherence.

Far from the source, the gravitational impulse can be represented by a superposition of plane waves with weights given by the Fourier transform of the signal envelope, $F_p + F_m$. As is well known, for waves propagating in the vacuum there exist only two independent amplitudes (Wald 1984), corresponding to two polarizations of transverse tidal oscillations. Let us focus on the pressure contribution (8) and consider the wave propagating in the z-direction. We can write the wave in the transverse-traceless (TT) gauge in the form

$$h_{TT}^{\mu\nu}(z,t) = \frac{1}{2}[\bar{h}_p^{11}(z,t) - \bar{h}_p^{22}(z,t)]\mathbf{e}_+^{\mu\nu}, \tag{12}$$

where the matrix $\mathbf{e}_+^{\mu\nu}$ is the unit tensor of the "plus" polarization. Formally, this sum is identically zero, as all the diagonal components $\bar{h}_p^{ii}$ in equation (8) are the same. Physically, however, we can notice that the amplitude (12) vanishes as a result of exact cancellation of two waves of precisely opposite polarizations, $h_{TT}^{\mu\nu} = A_+\mathbf{e}_+^{\mu\nu} + (-A_+)\mathbf{e}_+^{\mu\nu}$. This can happen only when the radiation generation is fully coherent. Since then no energy is emitted, we conclude that the energy must be radiated away by incoherent gravitational radiation.

In the case of violent explosion, we do not expect much coherence, in particular at short wavelenghts, as this would require suppression of any randomness in the formation of the fireball. Thus at high frequencies $\nu >> \bar{\nu}$, much higher than $\bar{\nu}$, a typical frequency $\bar{\nu} \sim 1/\Delta t \sim$1 Hz, we expect the gravitationl radiation to be incoherent. The presence of high frequencies (short wavelegths) depends on the time behaviour of the fireball formation. Very rapid formation could be approximated by an instantaneous process, with the time dependence of the source energy-momentum tensor (7) given by $F_p(t) = \Theta(t)$. The Fourier transform of $T^{\mu\nu}(\mathbf{r},t)$ is $S^{\mu\nu}(\mathbf{k},\omega) \sim i/(2\pi\omega)$. The corresponding energy distribution of the emitted radiation in frequency and angle is then (Adler & Zeks 1975)

$$\frac{dE}{d\Omega d\omega} \sim \omega^2 S_{ik}^*(\mathbf{k},\omega)S^{ik}(\mathbf{k},\omega) \sim const. \tag{13}$$

Clearly, the $F_p(t) = \Theta(t)$ behaviour is unrealistic, as the total energy of gravitational waves is infinite. It shows, however, that the more rapid the formation the higher frequencies are involved. Also, total energy radiated away as gravitational waves grows for more rapid formation processes (Adler & Zeks 1975). The physical function $F_p(t)$ provides a natural cut-off frequency.

When the short-wavelenght gravitational waves are produced abundantly, one should not regard the emitted gravitational radiation as propagating in a vacuum. In this case, as shown by Isaacson (1968), we can treat high-frequency perturbation in the geometrical optics limit,$h_{\mu\nu} \approx A_{\mu\nu} \exp(ik_\alpha x^\alpha)$ with suitably defined $A_{\mu\nu}$ and k. The energy of high-frequency gravitational waves should be included as a source term through an effective "graviton" stress-energy tensor

$$T^{\mu\nu}_{gw} = \Sigma_{\mathbf{k}} q(\mathbf{k})^2 k^\mu k^\nu, \tag{14}$$

where $q(\mathbf{k})^2 = \epsilon^2 A^*_{\mu\nu}(\mathbf{k}) A^{\mu\nu}(\mathbf{k}) c^4/32\pi G$. The low-frequency waves would thus satisfy the Einstein equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \frac{8\pi G}{c^4} T^{\mu\nu}_{gw} \tag{15}$$

which is not the empty space equation. Its form is the same as for gravitational waves associated with the electromagnetic shell considered in Section 3, when the photon stress-energy tensor is treated in the geometrical optics approximation, $T^{\mu\nu}_\gamma = \Sigma_{\mathbf{k}} q(\mathbf{k})^2 k^\mu k^\nu$ (Lindquist, Schwartz & Misner 1965). The same stress-energy tensor can also be used to describe the gravitational waves travelling together with the neutrino burst from a supernova explosion (Misner 1965), for neutrinos assumed to be massless particles.

Monopole gravitational waves propagating together with the radiation of massless fields are thus described in a unified way by equation (15), for any type of radiation. For massless fields, the wave vectors in the stress-energy tensor are null, $k_\mu k^\mu = 0$, and the curvature scalar in (15) is $R = 0$. When the radiation propagates in the z-direction, the only components of the stress-energy tensor are T_{00}, T_{0z}, and T_{zz}. Classification of polarization modes of gravitational waves, given by Eardley, Lee & Lightman (1973) shows that in this case the mode $\Phi_{22} = -R_{x0x0} - R_{y0y0}$ is non-zero, where R_{i0j0} are the so-called electric components of the Riemann tensor. This is a monopole breathing mode which can be identified with the $h^{11} + h^{22}$ wave of Eddington's classification (Eddington 1960). The monopole polarization differs from "plus" and "cross" polarizations of vacuum gravitational waves, and corresponds to a circle of test particles changing its radius and preserving circular shape in the plane perpendicular to the propagation direction of the wave.

6 DISCUSSION

The monopole radiation arising from the time-dependent pressure contribution to the gravitational mass would probe the general relativity sector not yet tested empirically, in which foundations of cosmology are rooted. This radiation would also encode valuable astrophysical information, transmitted directly from inside the relativistic fireballs formed in gamma-ray burst sources.

It is expected that in other phenomena involving relativistic fireballs, monopole gravitational waves are also emitted. In supernova explosions, high-pressure neutrino fireballs are formed, which would emit gravitational waves in a very similar manner to the plasma fireballs discussed here. One should keep in mind that, in astrophysical phenomena where gravitational waves are thought to be produced, usually a lot of energy is radiated away by massless fields, photons, neutrinos, and high-frequency gravitons. This could make generation of pure vacuum gravitational waves with only "plus" and "cross" polarizations less frequent than expected and the real gravitational signal could involve a significant monopole Φ_{22} contribution.

The monopole polarization that we have discussed is the same as predicted in scalar-tensor theories. The structure of the Ricci tensor in equation (15), for gravitational waves in the radiation background, is the same as for vacuum gravitational waves in Brans-Dicke theory (Brans & Dicke 1961). The stress-energy tensor for photons, neutrinos, and Isaacson's effective stress-energy tensor for gravitons play the same role in equation (15) as the scalar field term in the Brans-Dicke theory for gravitational waves propagating in a vacuum. This fact would make testing Brans-Dicke theory more difficult.

When some major inhomogeneity is involved in the formation of the fireball, resulting in the anisotropy of pressure, then the diagonal stresses, B^{ii}, equation (6), can differ from one another, say $B^{11}_p > B^{22}_p$. A coherent TT-wave can then be emitted,

$$h^{11}_{TT} = -h^{22}_{TT} = \frac{1}{2}(\bar{h}^{11}_p - \bar{h}^{22}_p). \tag{16}$$

This amplitude is formally the same as that for a time-dependend mass quadrupole. The gravitational wave detector response to such a gravitational wave would be similar to that for quadrupole waves of the same amplitude and frequency.

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REFERENCES

Adler R. J., Zeks B., 1975, Phys. Rev. D, 12, 3007
Brans C., Dicke R. H., 1961, Phys. Rev., 124, 925
Carlip S., 1998, Am. J. Phys., 65, 409
Eardley D. M., Lee D. L., Lightman A. P., 1973, Phys. Rev. D, 8, 3308
Eddington A. S., 1960, The Mathematical Theory of Relativity. University Press, Cambridge, UK
Garnavich P. M. et al., 1998, ApJ, 509, 74
Goodman J., 1986, ApJ, 308, L47
Isaacson R. A., 1968, Phys. Rev., 166, 1272
Kobayashi S., Piran T., Sari R., 1999, ApJ, 513, 669
Landau L. D., Lifshitz E.M.,1975, The Classical Theory of Fields. Pergamon, Oxford, UK
Lindquist R. W., Schwartz R. A., Misner C. W., 1965, Phys. Rev., 137, B1364
Mac Fadyen A. I.,Woosley S. E., 1999, ApJ, 524, 262
Misner C. W., 1965, Phys. Rev., 137, B1360
Paczyński B., 1986, ApJ, 308, L43
Peebles P. J. E., 1993, Principles of Physical Cosmology. Princeton Univ. Press, Princeton, NJ

Tolman R. C., 1934, Relativity, Thermodynamics and Cosmology. Clarendon, Oxford, UK

Wald R. M., 1984, General Relativity. Univ. of Chicago Press, Chicago, IL

Whittaker E. T., 1935, Proc. Roy. Soc., A 149, 384